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Lattice simulations of the strange quark mass and Fritzsch texture

Biswajoy Brahmachari
Indiana University

Strange quark mass has been traditionally calculated using current algebra mass ratio

$$\frac{m_s}{m_u + m_d} = 12.6 \pm 0.5 \tag{1}$$

Equation (1) is evaluated using values of $(m_u + m_d)$. At the two-loop level of perturbative QCD calculations which include non-perturbative corrections up to dimension six one has the result

$$(m_u + m_d)(\text{ 1 GeV}) = 15.5 \pm 2.0 \text{ MeV} \tag{2}$$

So we obtain

$$\begin{aligned} m_s(1\text{ GeV}) &= 195 \pm 28 \quad \text{MeV} \\ m_s(2\text{ GeV}) &= 150 \pm 21 \quad \text{MeV} \end{aligned} \tag{3}$$

The ratio

$$\begin{aligned} &m_s(2\text{ GeV})/m_s(1\text{ GeV}) = 0.769 \\ &\text{for} \quad \alpha_s(m_Z) = 0.118 \end{aligned} \tag{4}$$

can be obtained by solving renormalization group equations.

A systematic uncertainty in this result remains in reconstruction of so called 'spectral function' from experimental data of resonances. When a different functional form of the resonance is adopted, and three loop order perturbative QCD theory is used one obtains

$$(m_u + m_d)\ (1\text{ GeV}) = 12.0 \pm 2.5 \text{ MeV} \tag{5}$$

With (5) and (1) one gets

$$m_s(1\text{GeV}) = 151 \pm 32 \text{ MeV}$$
$$m_s(2\text{GeV}) = 116 \pm 24 \text{ MeV} \quad (6)$$

H.Leutwyler, Phys.Lett. B378,313 (1996)

It has been remarked by Leutwyler that it is indeed difficult to account for vacuum fluctuations, or sea quark effects generated by quarks of small masses in perturbative QCD calculations. Thus, numerical simulations of strange quark mass on a lattice becomes rather attractive, especially if the simulation includes virtual light quark loop effects.

• Up and down type quarks differ only in the $U(1)_{em}$ quantum numbers in an effective theory where the gauge symmetry is $SU(3)_c \times U(1)_{em}$.

• We are describing the lattice in terms of a theory at the scale of a few GeVs where light quark masses are to be described in terms of observables relevant to their own scales, which are meson masses and decay constants.

• Lattice simulation determines m_s, $\frac{m_u+m_d}{2}$ and the lattice spacing a using three hadronic observables. They can be chosen, for example, M_π, M_{K^*}, and f_π. Scale a can also be taken as a function of some other observable, for example, it may be chosen as $a(M_n)$ or $a(M_\Delta)$ etc.

• how do we describe quark masses when the theory is living on a discritized lattice? Wilson-like fermions are, for example

$$a\ m_{bare} = log\ (1 + (1/2\kappa - 1/2\kappa_c)) \qquad (7)$$

In the continuum limit we have $a \rightarrow 0$, and there one gets the hopping parameter $\kappa = \kappa_c = 1/8$.

R.Gupta and T.Bhattachary Phys.Rev. D55, 7203(1997)

• To compare the result with experiment, one has to calculate the $\overline{MS}$ mass at a scale μ starting from the lattice estimate of the bare mass using, for example, the mass renormalization constant $Z_m(\mu)$ relating the lattice regularization scheme to the continuum regularization scheme.

• Final results of the physical quark mass for various definitions of the fermion on a lattice differ $O(a)$ among each other and one expects to get the same result of the physical quark mass in the continuum limit when $a \rightarrow 0$.

• Beyond the minimal lattice simulation of light quark masses using the heavy quark effective theory, the next step would be to incorporate sea quark effects. They are termed $n_f = 2$ unquenched lattice simulations.

• Bottom quark mass is in the range

$$4.1 < m_b(m_b) < 4.4 \quad \text{GeV} \qquad (8)$$

according to review of particle physics (PDG) tables

ref	$quenched$	$dynamical$	$spacing$	$m_s(MeV)$
A	yes		m_ρ	143 ± 6 & 115 ± 2
B	yes		m_{K^*}	130 ± 20
C	yes		m_{K^*}	122 ± 20
D	yes		m_{K^*}	111 ± 12
E	yes		m_ρ	110 ± 31
F	yes		m_ρ	108 ± 4
G	yes		$1P - 1S$	95 ± 16
A		yes	m_ρ	70 & 80
E		yes	m_ρ	68 ± 19
G		yes	$1P - 1S$	$54 - 92$

(A) CP-PACS Collaboration (K. Kanaya *et al.*), Nucl. Phys. Proc. Suppl. 73, 192 (1999).

(B) V. Gimenez *et al.*, Nucl. Phys. B540, 472 (1999).

(C) C. R. Allton *et al.*, Nucl. Phys. B489, 427 (1997).

(D) D. Becirevic *et al.*, Nucl. Phys. Proc. Suppl. 73, 222 (1999).

(E) R. Gupta and T. Bhattacharya, Phys. Rev. D55, 7203 (1997).

(F) M. Gockeler *et al.*, Nucl. Phys. Proc. Suppl. 73, 237 (1999); Phys. Rev. D57, 5562 (1998).

(G) B. J. Gaugh *et al.*, Phys. Rev. Lett. 72, 1622 (1997).

Theoretically, one re-expresses the bottom quark mass in terms of parameters of the Minimal Supersymmetric Standard Model(MSSM). The tree level contribution which is related straight to the Yukawa texture, and the one loop contribution due to the dominant gaugino loop can be accounted individually. Then one can write down the relation

R.Rattazzi, U.Sarid and L.J.Hall. SU-ITP-94-15, May 1994

$$\begin{aligned} m_b &= m_b^{texture} + m_b^{SUSY} \\ &= h_b \ \frac{V_F}{\sqrt{2}} \ \cos \ \beta + m_b \ \frac{8}{3} \ g_3^2 \ \frac{\tan\beta}{16 \ \pi^2} \ \frac{m_{\tilde{g}} \ \mu}{m_{eff}^2} \end{aligned} \qquad (9)$$

Here $m_{\tilde{g}}$ is the gluino mass μ is the μ parameter and m_{eff} is averaged supersymmetry breaking mass scale.

We will discuss a scenario where the first term of the RHS of (9) comes from diaginalizing a Fritzsch Yukawa texture.

• Suppose in a two generation case rotation angles of the up and the down sectors are θ_u and θ_d. Then of the combined quark mixing matrix

S.Weinberg, Trans. NewYork Acad.Sci. 38, 185 (1977)

$$V = O_u {O_d}^\dagger \quad \longrightarrow \quad \theta_c = \theta_u \pm \theta_d \qquad (10)$$

• the ratio of the masses of the first and the second generation satisfies well the relation

$$\tan\theta_c = \sqrt{\frac{m_d}{m_s}}. \qquad (11)$$

Fritzsch mass matrices can be thought of as a set of mass matrices which generalizes (11) to the following form

$$\theta_c = \theta_d \pm \theta_u \;\; \longrightarrow \;\; \tan^{-1}\sqrt{\frac{m_d}{m_s}} \pm \tan^{-1}\sqrt{\frac{m_u}{m_c}} \quad (12)$$

In the three generation case Fritzsch textures for up and down sectors are given by

$$M_U = \begin{pmatrix} 0 & ae^{ir} & 0 \\ ae^{ir'} & 0 & be^{ih} \\ 0 & be^{ih'} & ce^{iq} \end{pmatrix}$$
$$M_D = \begin{pmatrix} 0 & Ae^{iR} & 0 \\ Ae^{iR'} & 0 & Be^{iH} \\ 0 & Be^{iH'} & Ce^{iQ} \end{pmatrix} \quad (13)$$

weak mixing matrix is expressed as

$$O_U \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix} {O_D}^{-1} \quad (14)$$

where the phases are

$$\begin{aligned} \sigma &= (r-R) - (h-H) - (h'-H') + (q-Q) \\ \tau &= (r-R) - (h'-H') \end{aligned} \quad (15)$$

In (14) O_U and O_D diagonalizes M_U and M_D in the limit when all the phases vanish.

• We will use the approximation of strongly hierarchical eigenvalues.

$$\begin{pmatrix} 1 & -\nu_1+\mu_1 e^{i\sigma} & \mu_1(\nu_2 e^{i\sigma}-\mu_2 e^{i\tau}) \\ -\mu_1+\nu_1 e^{i\sigma} & \mu_1\nu_1+\mu_2\nu_2 e^{i\sigma}+e^{i\sigma} & \nu_2 e^{i\sigma}-\mu_2 e^{i\tau} \\ \nu_1(\mu_2 e^{i\sigma}-\nu_2 e^{i\tau}) & \mu_2 e^{i\sigma}-\nu_2 e^{i\tau} & \mu_2\nu_2 e^{i\sigma}+e^{i\tau} \end{pmatrix}$$

where

$$\mu_1=\sqrt{\frac{m_u}{m_c}} \quad \mu_2=\sqrt{\frac{m_c}{m_t}} \quad \nu_1=\sqrt{\frac{m_d}{m_s}} \quad \nu_2=\sqrt{\frac{m_s}{m_b}} \tag{16}$$

K.S.Babu and Q.Shafi, Phys.Rev. D47,5004 (1993)

• It is easy to check that Fritzsch relations make the top quark mass too light to be experimentally true ($\sim$ 100 GeV).

• If the flavor symmetries were exact only above the GUT scale, could a miracle of renormalization group evolution of the masses and mixing angles make the Fritzsch relations valid at low energy ?

one can choose before making any other computation

$$\sigma \sim \tau \sim -\frac{\pi}{2} \tag{17}$$

Let us set our notations of mixing angles, the non-removable phase and eigenvalues of the Yukawa matrices. We adopt the parameterization

$$\begin{pmatrix} s_1 s_2 c_3 + c_1 c_2 e^{i\phi} & c_1 s_2 c_3 - s_1 c_2 e^{i\phi} & s_2 s_3 \\ s_1 c_2 c_3 - c_1 s_2 e^{i\phi} & c_1 c_2 c_3 + s_1 s_2 e^{i\phi} & c_2 s_3 \\ -s_1 s_3 & -c_1 s_3 & c_3 \end{pmatrix}. \tag{18}$$

In this parameterization eigenvalues y_i of the Yukawa textures, three CKM mixing angles and the CP violating phase ϕ satisfy the following renormalization group equations S.Naculich, Phys. Rev. D48, 5293 (1993)

$$
\begin{aligned}
16\pi^2\frac{d}{dt}\phi &= 0 \quad , \\
16\pi^2\frac{d}{dt}\ln\tan\theta_1 &= -y_t^2\sin^2\theta_3 \quad , \\
16\pi^2\frac{d}{dt}\ln\tan\theta_2 &= -y_b^2\sin^2\theta_3 \quad , \\
16\pi^2\frac{d}{dt}\ln\tan\theta_3 &= -y_t^2-y_b^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_u &= -c_i^u g_i^2+3y_t^2+y_b^2\cos^2\theta_2\sin^2\theta_3 \quad , \\
16\pi^2\frac{d}{dt}\ln y_c &= -c_i^u g_i^2+3y_t^2+y_b^2\sin^2\theta_2\sin^2\theta_3 \quad , \\
16\pi^2\frac{d}{dt}\ln y_t &= -c_i^u g_i^2+6y_t^2+y_b^2 cos^2\theta_3 \quad , \\
16\pi^2\frac{d}{dt}\ln y_d &= -c_i^d g_i^2+y_t^2\sin^2\theta_1\sin^2\theta_3+3y_b^2+y_\tau^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_s &= -c_i^d g_i^2+y_t^2\cos^2\theta_1\sin^2\theta_3+3y_b^2+y_\tau^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_b &= -c_i^d g_i^2+y_t^2\cos^2\theta_3+6y_b^2+y_\tau^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_e &= -c_i^e g_i^2+3y_b^2+y_\tau^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_\mu &= -c_i^e g_i^2+3y_b^2+y_\tau^2 \quad , \\
16\pi^2\frac{d}{dt}\ln y_\tau &= -c_i^e g_i^2+3y_b^2+4y_\tau^2 \quad .
\end{aligned} \tag{19}
$$

α_s	$\tan\beta$	$m_s(2\ \text{GeV})$
0.118	2	59.90 MeV
0.118	10	61.52 MeV
0.118	20	63.05 MeV
0.118	30	66.90 MeV

• Implications of results of $n_f = 2$ unquenched lattice simulations of the strange quark mass in the context of the Fritzsch texture are studied.

• Fritzsch texture demands a large Yukawa contribution to the bottom quark mass. This is partially canceled by the supersymmetric loop corrections.

• We conclude that original Fritzsch texture is consistent with experimental data if it holds at the GUT scale.